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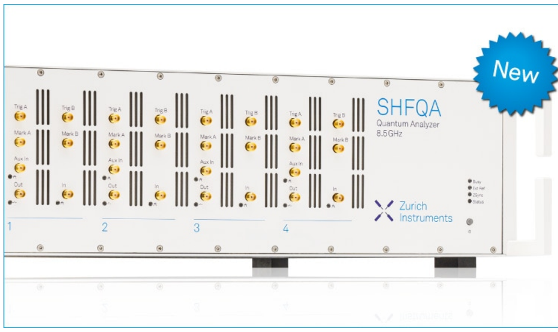

Mathematical model of the competition life cycle under limited resources conditions: problem statement for business community

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Abstract. Present study is devoted to the development of competition life cycle mathematical model in the closed business community with limited resources. Growth of each agent is determined by the balance of input and output resource flows: input (cash) flow W is covering the variable V and constant C costs and growth dA/dt of the agent's assets A. Value of V is proportional to assets A that allows us to write down a first order non-stationary differential equation of the agent growth. Model includes the number of such equations due to the number of agents. The amount of resources that is available for agents vary in time. The balances of their input and output flows are changing correspondingly to the different stages of the competition life cycle. According to the theory of systems, the most complete description of any object or process is the model of its life cycle. Such a model describes all stages of its development: from the appearance ("birth") through development ("growth") to extinction ("death"). The model of the evolution of an individual firm, not contradicting the economic meaning of events actually observed in the market, is the desired result from modern AVMs for applied use. With a correct description of the market, rules for participants' actions, restrictions, forecasts can be obtained, which modern mathematics and the economy can not give.

INTRODUCTION

Scientific and technological progress and economic development in the years that have passed since the emergence of the theory of economic development of J. Schumpeter have shown that the coexistence of agents on the same market with different competitive behavior strategies is one of the important phenomena of the modern economy.

According to the theory of systems, the most complete description of any object or process is the model of its life cycle. Such a model describes all stages of its development: from the appearance ("birth") through development ("growth") to extinction ("death"). The model of the evolution of an individual firm, not contradicting the economic meaning of events actually observed in the market, is the desired result of modern AVMs for applied use [1].

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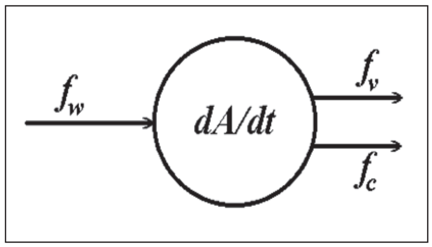
The work is devoted to the development of a mathematical model ab initio describing all stages of competition for a limited resource in the market. It is based on the description of the functioning of one economic agent as an open system.

With the correct description of the market, participants' rules of action, limitations, forecasts, which modern mathematics and economics can not give.

INTERACTION OF AN ECONOMIC AGENT WITH THE EXTERNAL ENVIRONMENT

The basis of the mathematical model is the representation of the economic system as an open system, the development of which is based on the interaction of input and output resource flows: the input resource flow fw (for example, gross revenue) is distributed to cover the variables fv and fixed costs fc, and the increase dA / dt of own assets A Economic system.



I = dA/dt + C

I – Agent income, *dA/dt* – Agent asset growth, C – costs

FIGURE 1. Interaction of an economic agent with the external environment [2].

The condition of closure of the system

$$\sum A_t(t) + L(t) = const, npu\ \forall\ t$$

In the presence of an external source of the resource, the input W of the resource-the object of competition-enters the agent's input (in general, this is the solvent demand of customers, which becomes the agent's income). Of all this flow, the agent transforms as much as his production capabilities allow - that is, Assets A. The potential volume of the transformation P in the simplest case is directly proportional to A: P = pA. The condition of closure of the system.

Since the obtained expressions are valid for any agent, the behavior of the aggregate of agents will be described by the solution of the following system of N (t) differential equations (N (t) is the total number of agents in the system in the time interval [0-t]. In Fig. 2 presents a system of differential equations describing the mathematical model of the life cycle of a system with n agents.

$$\begin{cases} dA_1/dt = fp_1(A_1(t), AStr_1(t), t) - fv_1(fp_1(t), fW_1(t), A_1(t), AStr_1(t), t) - fm_1(A_1(t), AStr_1(t), t) \\ dA_2/dt = fp_2(A_2(t-\tau), AStr_2(t-\tau), t) - fv_2(fp_2(t-\tau), fW_2(t-\tau), A_2(t-\tau), AStr_2(t-\tau), t) - fm_2(A_2(t-\tau), AStr_2(t-\tau), t) \\ dA_3/dt = fp_3(A_3(t-2\tau), AStr_3(t-2\tau), t) - fv_3(fp_3(t-2\tau), fW_3(t-2\tau), A_3(t-2\tau), AStr_3(t-2\tau), t) - fm_3(A_3(t-2\tau), AStr_3(t-2\tau), t) \\ \dots\dots\dots \\ dA_n/dt = fp_n(A_n(t-(n-1)\tau), AStr_n(t-(n-1)\tau), t) - fv_n(fp_n(t-(n-1)\tau), fW_n(t-(n-1)\tau), A_n(t-(n-1)\tau), AStr_n(t-(n-1)\tau), t) - fm_n(A_n(t-(n-1)\tau), AStr_n(t-(n-1)\tau), t) \end{cases}$$

FIGURE 2. General numerical model of CLC.

fvi = *fvi(fpi(t), fWi(t), Ai(t), AStri(t), t)* – variable costs;
fmi= *fmi(Ai(t), AStri(t), t)* – fixed costs;
fpi = *fpi(Ai(t), AStri(t), t)* – potential capacity of resource conversion by agent;
fWi = *fWi(Ai(t), AStri(t), L(t), STR(t), t)* – resource flow to agent;

The solution of the system of differential equations is replaced by numerical calculation in the medium of cellular automata (CA). The peculiarity of models in a spacecraft environment is that only initial and boundary conditions are specified, as well as rules for the interaction of the elements of the system, after which it observes the process of self-development of the system [3-4]. The possibilities of cellular automata as a computing environment make it possible to realize the evolution of complex hierarchical systems with a large number of elements that interact nonlinearly with each other. At the same time, an increase in the number of agents in the model does not lead to a significant increase in the complexity of the problem being solved, as is the case in systems of differential equations).

An imitation model of the life cycle of the economic system has been developed. The described CLC model is an extension of the known zero-sum game model. The total amount of resources consumed by agents remains constant, and the exchange of resources leads to its redistribution among agents. The main feature of the model is that external agents are included in the review. In this case, the aggregate of agents is an "open" system that dynamically interacts with the external environment.

Changing over time the amount of free resource in the system changes the balance of the input and output streams of agents, which corresponds to different stages of the life cycle of competition.

During the calculation, the agent is visually viewed on the computer screen. The influence of positive feedback on the "collision" between large and small firms is well traced (most of them entered the market earlier, a small one - later, because of the same growth, the size reflects the time of the firm's work in the market): the relative and absolute growth rate of a large Firm more than a small. The transition from one stage of the life cycle to another is also well traced on the model (Fig. 3).

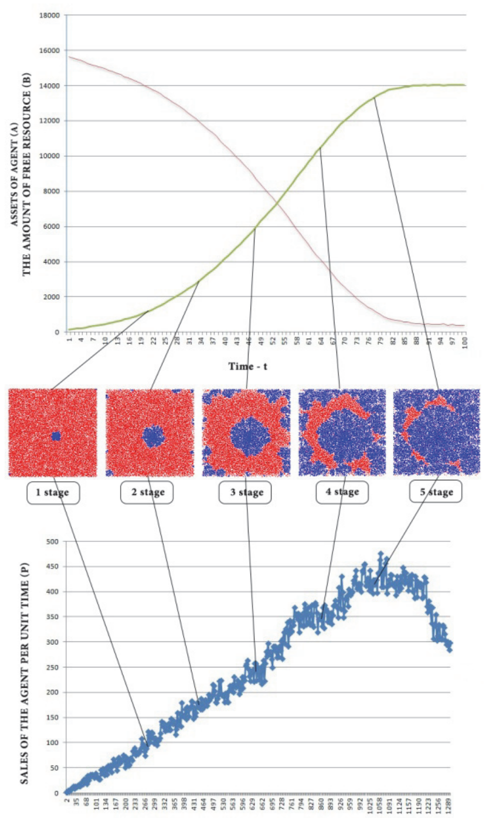


FIGURE 3. Stages of the life cycle of the economic agent in the CA

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The model captures the development of the agent in digital and graphical forms.

The resource consumption rule (with variable costs) means that the element *Lij* is attached with some probability $\rho kn(t)$ to the growing fixed agent Sn, as soon as it is $k \geq 1$ in some neighborhood of the *ij*-element:

$Xij(t) \in L \rightarrow Xij(t+1) \in Sn$ with probability $\rho kn(t)$, если $\{Mij(t) \cap Sn(t)\} \neq \varnothing$,

Where $Xij(t)$ и $Xij(t+1)$ - State of the ij-element of the matrix at two consecutive times; $Mij(t) = \{Xi+1,j(t);Xi-1,j(t);Xi,j+1(t);Xi,j-1(t)\}$ – Its vicinity Moore (in general, any other). $\rho 1n(t) < \rho 2n(t) < \rho 3n(t) < \rho 4.n(t)$.

The emergence of new aggregation centers (individual S-elements or their microclusters) comes from L-elements with some given probability, which can be a function of time. For the model to work:

Control parameters

- Initial amount of free resource
- Specific values of constant and variable costs of the agent
- Mutual location of agents

The response of the system (in time)

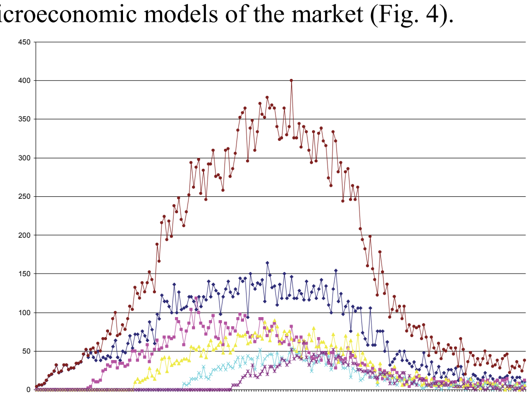
- Assets of each agent
- Sales volume of each agent
- Structural characteristic of the agent
- The amount of resources that is available for agents vary in time.

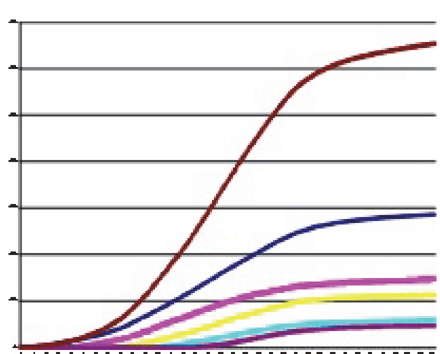
Consumption of the resource (joining elements $\boldsymbol{L}$ to the agent-aggregate $\boldsymbol{S^n}$) leads to an increase in its assets dA_n/dt ($A_n(t) \equiv \boldsymbol{S^n}(t)$ – number of elements in the unit, their separation - to a decrease). The coordination number k of each ij-element from $\boldsymbol{S^n}$ characterizes the local structural characteristic of assets $AStr_n(t)$. The value of variable costs $f_{vn}(A_n(t), AStr_n(t), t) > 0$ is determined by the values of probabilities $(1-\rho_k^n(t)) > 0$ – parts of resource elements that have touched the agent aggregate, but did not form a connection with it. Maximum possible amount of consumed resource $f_{pn}(AStr_n(t), A_n(t), t)$ is determined by the agent's available free surface: ($A_n(t)$) and its structure $AStr_n(t)$.

Flow $f_{Wn}(A_n(t), AStr_n(t), \boldsymbol{L}(t), STR(t), t)$ free resource to each agent is determined $\boldsymbol{L}(t)$, Function $STR(t)$ - mutual spatial arrangement $\boldsymbol{L}(t)$ и $\boldsymbol{S}(t)$, and as well the aggregate of the absolute value of the assets of the agent $A_n(t)$ and its structural characteristic $AStr_n(t)$, providing effective cross-section of the agent in the resource flow.

Specific fixed costs $f_{mn}(A_n(t), AStr_n(t), t)$) are determined by the values $P_k^n(t)$, taking into account the structure of assets $AStr_n(t)$ across k. Appearance ($N(t)$) and the localization of new agents is given by some function of time. Dependence of model parameters on time provides the possibility of external system control. For economic systems, one of the conditions is $Pkn(t) \neq Pkr(t)\ \forall\ n,r / n,r \in \{N(t)\}$ – there are no identical agents in the system.

The above calculations show the possibility of developing a complete model of macroeconomic cycles based on microeconomic models of the market (Fig. 4).





Volume of sales Assets

FIGURE 4. Trajectories of CLC

The balances of their input and output flows are changing correspondingly to the different stages of the competition life cycle. An imitation model of the life cycle of the economic system has been developed. The described CLC model is an extension of the known zero-sum game model. The total amount of resources consumed by agents remains constant, and the exchange of resources leads to its redistribution among agents. The main feature of the model is that external agents are included in the review. In this case, the aggregate of agents is an "open" system that dynamically interacts with the external environment.

During the calculation, the market section is visually observed on the computer screen. The influence of positive feedback in the "collision" between large and small firms is well traced (the large one came to the market earlier, the small one - later, because of the same growth rate, the size reflects the firm's operating time on the market): the relative and absolute growth rate of a large firm is greater than the small one. The transition from one stage of the CCW is also well traced (fig. 5).

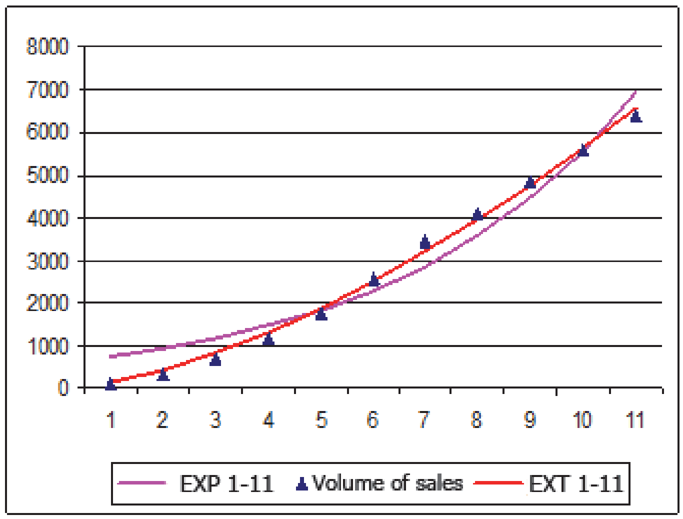


FIGURE 5. Competition strategy approach

Approximation of the experimentally obtained data at the stage of accelerated growth of the life cycle of the economic agent with the help of power and exponential functions showed that the accuracy is higher for the power function than for the exponential function. The exponential growth curve is borrowed from the biological approach [5], when it comes to the development of a population of biological organisms and less suitable for economic agents.

TABLE 1. Approximation of "VimpelCom" graphs

Exponent 1-11		Extent 1-11	
b	606,32952	d	147,4003
a	0,2214878	c	1,581513
SSE	2354123,7	SSE	205727,6
R^2	0,9568219	R^2	0,99624
MSD	709795	MSD	62029,19

Approximation of the exponential curve and the study of A.Yu. Yudanov [6-7] of VimpelCom's growth dynamics confirmed the hypothesis put forward (Table 1).

CONCLUSION

The experimental model of the LC of the market and agents is presented. Changing the control parameters allows you to obtain various curves of the LC market and individual agents.

The boundaries of the individual phases of the LC are clearly defined.

Model calculations allow testing hypotheses about the influence of certain conditions on the parameters of curves analytically describing the market development of the market and individual agents

On the basis of the model, it is possible to analyze the individual stages of the development of agents for testing hypotheses of development patterns.

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REFERENCES

1. S. Lapshina, D. Berg , A. Grokhovsky, “Theoretical bases of the analysis of competitive behavior strategies of agents in the market”, *Journal of Economic theory*, 4. pp. 5-23 (2007).
2. V. Popkov, D. Berg, “Empirical identification of competitive strategies: Russian Bank System”, In Empirical Science of Financial Fluctuations: The Advent of Econophysics, ed. H.Takayasu. Tokyo: Springer-Verlag. pp. 331-340 (2001).
3. D.B. Berg, K.A. Beklemishev, A.N. Medvedev and M.A. Medvedeva, “Modeling of the competition life cycle using the software complex of cellular automata PyCAlab”, AIP Conference Proceedings 1690, 030003; doi: 10.1063/1.4936702 (2015).
4. T. Toffoli, N. Margolus, “Cellular automata machines: A new environment for modeling, Massachusetts”, MIT Press (1990).
5. A. Yudanov, “The types of competitive strategies: a “biological” approach to the classification of companies”, *World economy and international relations*, 10, pp. 51-63 (1990).
6. A. Yudanov, “High-growth LMT firms and the evolution of the Russian economy”, Knowledge-Intensive Entrepreneurship in Low-Tech Industries, May 30, Pages 117-137 (2014).
7. M.A. Medvedeva, R.H. Davletbaev, D.B. Berg, J.J. Nazarova and S.S. Parusheva, “Industrial entrepreneurial network: Structural and functional analysis”, AIP Conference Proceedings, 1789, 020018, http://doi.org/10.1063/1.4968439 (2016).